APhEx 7, 2013 (ed. Vera Tripodi)
Redattore: Daniele Santoro

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PORTALE ITALIANO DI FILOSOFIA ANALITICA
GIORNALE DI FILOSOFIA
NETWORK
N°7 GENNAIO 2013

I n t e r v i s t e

Conversation with Stewart Shapiro

by Sebastiano Moruzzi and Andrea Sereni

1. *Prof. Shapiro, you are one of the leading philosophers in contemporary philosophy of mathematics and philosophy of logic. Beside being the author of some essential books and a vast number of papers on these fields, you are one of the main proponents of the view known as structuralism, and you have suggested original views concerning, for instance, second-order logic and vagueness. We assume that your interest in these issues comes from your studies in Buffalo with John Corcoran. Would you help our readers understanding how you decided to get involved professionally in these areas of philosophy, which colleagues or philosophers inspired or guided you in making this choice, and why you decided to be a philosopher in the first place?*

SS. Although it has been a long time, and my memory is not very reliable, I'll begin with the last question. Even as a small boy, I was attracted to mathematics – I was something of a nerd. Way back in junior high school, I stumbled onto some popular works describing advanced mathematics, and was immediately hooked. I was profoundly fascinated by the notion of rigorous proof, with the very idea that through careful reasoning, one can (apparently) decide the truth of some proposition, putting it beyond all doubt. My seventh-grade mathematics teacher, Samuel Traficant, was most encouraging, giving me literally hours of his time after school. During high school, I attended a National Science Program in mathematics, held each summer here at Ohio State University. It was directed by Arnold Ross, a wonderful teacher and leader. At this program, there was a course in mathematical logic, taught by Ivo Thomas. I was then introduced to Gödel's completeness and incompleteness theorems, and fell in love with them. When I enrolled at Case Western Reserve University, to begin my university experience, I already knew that I wanted to study logic. I was surprised and delighted to learn that it was possible (at least in theory) to devote one's career to this. At Case, I was also blessed with wonderful teachers, including Ray Nelson and Howard Stein, introducing me to various aspects of mathematical logic and set theory. My interests also began to turn philosophical, and toward the end of my college career, I managed a double major, in mathematics and philosophy. I then entered the Ph.D. program in mathematics at the State University of New York at Buffalo, in 1973, but switched to philosophy a year later. My ability to find outstanding teachers continued there. First and foremost, of course, was my advisor, John Corcoran. At that time, Buffalo was a veritable powerhouse in logic – covering philosophy, mathematics, and computer science. The list includes John Kearns, Nicolas Goodman, John Myhill, John Case, Richard Vesley, Harvey Friedman, Leo Harrington, and Thomas Jech. There was also a wonderful spirit of cooperation among the logicians in the various departments. It was a great place to be trained in logic, both philosophical and mathematical. Of course, the subject matter itself is important to me. I am attracted to the fact that logic is a central branch of both mathematics and philosophy. One can, it seems, shed light on deep and interesting philosophical problems, concerning epistemology, language, and even metaphysics, through rigorous, formal techniques. Here is one place where certain formal theorems have profound ramifications for central philosophical issues.

2. *In many introductory books, the philosophy of mathematics is presented as an inquiry into the foundations of mathematics. Often, foundations are conceived in the way in which e.g. Frege, Russell and other figures of the main traditional schools conceived of them: as a way of securing our mathematical knowledge on unshakeable grounds. Traditional wisdom - especially in the Quinean footsteps - has it that first-order language is the privileged language in which such foundations should be framed, for higher-languages are seen as already mathematical in character. In your book Foundations without Foundationalism (Shapiro, 1991) you forcefully reacted to this picture. Could you tell us why you think that second-order logic is a suitable framework for a philosophical account of mathematics?*

SS. My attitude on these matters is summed up nicely by Jon Barwise: "As logicians, we do our subject a disservice by convincing others that logic is first-order and then convincing them that almost none of the concepts of modern mathematics can really be captured in first-order logic" (Barwise, 1985, p.5). As you note, sometimes the foundational project is described as one of putting mathematics on an absolutely secure, extra-mathematical foundation. The goal, supposedly, is to show, on some sort of non-mathematical ground, that mathematics *is* absolutely certain. I think it is generally (but not universally) agreed that all such projects have failed. As I understand it, it was not Frege's goal to remove any lingering doubts about the basic principles of mathematics, say arithmetic. Rather, he wanted to know *how* these truths are justified. It is an explanatory project. As he once put it,

> The aim of proof is, in fact, not merely to place the truth of the proposition beyond all doubt, but also to afford us insight into the dependence of truths upon one another. After we have convinced ourselves that a boulder is unmoveable, . . . there remains the further question, what is it that supports it so securely? (Frege, 1884, § 2)

I take it as a working hypothesis that mathematicians do have a grasp on notions like finitude, minimal closure, well-foundedness, and the like. I am not looking to criticize or justify this assumption. I take such knowledge as given, and insist that it is legitimate to focus on semantic frameworks that capture this assumption. So my own orientation is somewhat different from Frege's.

I do not wish to claim, however, that there is no role for first-order logic, nor for systems that are restricted in that way (nor, for that matter, for intermediate logics). There are many legitimate facets, intellectual goals, and enterprises that go by the name of logic, both in mathematics and in philosophy.

3. *In 1997, you published Philosophy of Mathematics: Structure and Ontology (Shapiro, 1997). In this book you presented a structuralist position, that accounted for pure*

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mathematics while steering clear of traditional platonism in both its logicist (and neo-logicist) version, and from its Gödelian brand. According to your ante rem structuralism, mathematics is the science of structures, and structures are conceived as self-standing platonic objects. You point to insights from Hilbert, Dedekind, and Benacerraf in formulating your view. Could you explain what you take the advantages of your view to be with respect to its rivals, both in the platonist and in the nominalist camps? Do you still adhere to your early formulation of your view, or do you feel that something should be modified on the basis of subsequent debate?

SS. Well, to begin, I am not sure that my view is that much of a departure from traditional platonism, at least on ontological and metaphysical matters. As you note, *ante rem* structures, as I seem them, are self-standing *platonic* objects. As the name indicates, *ante rem* structures exist independent of any systems that may exemplify them. In that respect, they are like platonic Forms. Because of this, Fraser MacBride dubs my view “old news” (MacBride, 2005, p. 584). I suppose he is right about that. In Philosophy, platonism is about as old as it gets.

On the metaphysical front, the central innovation behind *ante rem* structuralism concerns the very notion of an object. The idea is that a place in an ante rem structure is a bona fide object, and thus a legitimate range for bound variables. There are interesting questions concerning the extent to which this structural notion of object applies to ordinary objects. Fascinating issues concerning the nature of language and reference lie in the vicinity.

Probably the deepest and most difficult problems with traditional platonism concern epistemology. How can we know about platonic objects (whether they be objects as traditionally conceived, *ante rem* structures, or places in *ante rem* structures)? How can we have knowledge *of* such things, and knowledge *that* certain facts about them are true? How can we have any confidence that our beliefs about such objects are true? In the book, I argue (in Chapter 4) that *ante rem* structuralism has a line on epistemology, one that is more plausible than the traditional platonic one of postulating a faculty of special access to the platonic realm.

The view also accounts for the objectivity of mathematics, and, to some extent, for the applications of mathematics. Most importantly, it explains why, in mathematics, the nature of the individual objects does not matter. Only structure does. And it allows one to take the statements of mathematics at face value.

To tie this into the previous question, concerning foundational studies, I am not claiming that I can prove, on *a priori*, deductive grounds, from uncontroversial, non-mathematical premises, that ordinary mathematicians have knowledge of *ante rem* structures (and their places). Skepticism and irrealism are probably impossible to refute with apodictic certainty. It is more of an inference to the best explanation. I claim that my view – ontology and epistemology combined – provides a satisfactory account of mathematics and its place in our intellectual lives. The account should be judged, on holistic grounds, against other philosophies of mathematics.

You ask if my views have evolved. I think I still adhere to the early formulation of the view, at least in broad outline. Since the publication of the book, I have thought a lot more about the role of identity, and the practice of identifying mathematical objects with other mathematical objects, for various purposes. This appears in several publications, and results in some re-formulation of the underlying realism. The matter of identity also bears on my thinking in the philosophy of language generally.

I might add that, in the book, I suggest that there is some sort of tradeoff between *ante rem* structuralism and its eliminative rivals. The strengths and problems of the views are analogous to each other, and they are roughly on a par with each other. I still hold that, with even more conviction.

4. *The philosophy of mathematics seems to have experienced a sort of a renaissance in the past two decades. Many views have been suggested and refined (e.g. neo-logicism and structuralism), and many arguments have been explored in the realism vs. anti-realism debate (e.g. Field's nominalism, fictionalism, or the indispensability argument), as is well witnessed from the papers collected in the* Oxford Handbook of the Philosophy of Mathematics and Logic *that you edited in 2005 (Shapiro, 2005). Recently, there have been many connections between traditional inquiries in the philosophy of mathematics and issues in the philosophy of science - such as explanation - and in the cognitive sciences - such as the cognitive basis of our mathematical competencies. Moreover, vast attention is being given by philosophers to actual mathematical practice. Some may see these recent developments as a symptom that the philosophy of mathematics should abandon its foundational chimeras, and rather look, in a naturalist spirit, at the place mathematics has in the empirical world. Do you think that there is any sense (or maybe more than one) left in which an inquiry into the foundations of mathematics is still meaningful?*

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SS. I don't suppose I can just say "yes", and go on to the next question. The work you mention is terrific, and sheds a lot of light on the nature of mathematics, and the role of mathematics in the overall intellectual enterprise including, of course, empirical science. But I do not see how the enquiries you highlight somehow preclude or obviate the original problems and issues. In particular, I can't see how such the new foci somehow preclude asking the more global questions in traditional philosophy of mathematics.

As I put it in the structuralism book, some of the traditional questions are: "What is the subject matter of mathematics? What is the relationship between the subject matter of mathematics and the subject matter of science which allows such extensive application and cross-fertilization? How do we manage to do and know mathematics? How can mathematics be taught? How is mathematical language to be understood? In short, the philosopher must say something about mathematics, something about the applications of mathematics, something about mathematical language, and something about ourselves." I do not see how the profound work you mention does away with the urgency of these questions. In many cases, the work in question contributes to answers to these original questions, and sometimes it shows that the questions themselves are more subtle and nuanced than we might have thought.

5. *Your interests in the philosophy of mathematics is closely connected with your works in logic and the philosophy of logic. Logic is often intended as the study of the notion of logical consequence. You have advocated a second-order characterization of this notion. Second-order logic is more powerful than first-order one in expressive resources - for example, it can express notions like finiteness or countability - but some*

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philosophers and logicians have stressed some allegedly critical shortcomings of this logical theory. To mention some of these, the incompleteness of second-order logic with respect to standard semantics and its commitments to theoretical notions that seem to go beyond pure logic somehow disqualify it. Could you say why you think second-order logic is a philosophically legitimate way to characterize logical consequence?

SS. Almost from the start of my career, I have been arguing that there is no, single, monolithic notion of validity and of logical consequence. These are more like cluster concepts that can be sharpened in various ways. First-order logic is a good model of the deductive aspects of consequence, related to correct inference and consistency in reasoning. Second-order logic, and other model-theoretic studies, focus more on the semantic side. As above, I take it as a sort of data point that mathematicians do understand – univocally – certain notions, such as finitude, minimal closure, well-foundedness, countability, and the like. So any semantics-based logic should recapitulate such notions. Any logic that is aimed at the semantic aspects of the intuitive notion can *presuppose* that these notions are unequivocal. Of course, this is not to say that logic somehow *justifies* our belief that these notions are univocal, and well-understood. It is part of the anti-foundationalist approach that there is no non-mathematical justification to be had.

I am fond of quoting Alonzo Church:

> [O]ur definition of the [standard second-order] consequences of a system of postulates … can be seen to be not essentially different from [that] required for the … treatment of classical mathematics … It is true that the non-effective notion of consequence, as we have introduced it … also in classical mathematics, especially classical analysis. (Church, 1956, 356n)

The legitimacy of second-order languages, understood with (so-called) standard semantics, is of-a-piece with the thesis that we understand ordinary mathematical discourse. Second-order consequence is no worse than standard mathematics. And no better.

6. *In the debate in philosophy of logic we can find two main positions with respect to the analysis of logical consequence: deductivists hold that the best way to analyze this notion is by formalizing it through a calculus. Natural deduction has been prominent in this approach: the primitive rules of natural deduction systems are taken as representing the basic patterns of our deductive modes of reasoning. In contrast with the deductivist approach, model theorists have argued for the semantic approach which takes as primary the semantics instead of the calculus. You have suggested (Shapiro 2007) that it is perhaps misleading to view these approaches as excluding one another, that they are both philosophically legitimate and useful ways to represent formally our notion of logical consequence. Could you explain your reasons for such a pluralistic stance?*

SS. The upshot, again, is that there just is no single notion of logical consequence. There are deductive notions, semantic notions, epistemic notions, modal notions, and perhaps even others. Each camp focuses on one of these or, perhaps better, one aspect of the intuitive notion. Deductivists and model-theorists each present a sharpening of the amorphous, intuitive notion. My claim is that more than one such sharpening is legitimate and, indeed, useful.

Both the deductivist and the model-theoretic orientations suggest interesting questions concerning the meanings of the logical terms. The answers to those questions seem to be odds with each other. As you note, the inferentialist holds that the meaning of at least logical terminology is given by the rules of natural deduction systems. Michael Dummett, for example, writes:

> Gerhard Gentzen, who, by inventing both natural deduction and the sequent calculus, first taught us how logic should be formalised, gave a hint how to do this, remarking without elaboration that 'an introduction rule gives, so to say, a definition of the constant in question', by which he meant that it fixes its meaning. (Dummett, 1991, p. 251)

Model-theorists, on the other hand, tend to think of meaning as given by truth-conditions. In particular, the meaning of a logical term is the role it plays in the truth-conditions of sentences that contain it. These seem like incompatible claims (putting aside differences within each camp concerning what the proper rules and truth-conditions are). How are these matters to be adjudicated? Are those empirical claims about natural language words? If so, should we logicians resort to the techniques of empirical linguistics?

I am not sure what to make of these meta-semantic matters. We are clearly broaching deep issues concerning the nature of language and meaning. My eclectic orientation toward logic gives rise to an eclectic orientation toward the entire enterprise of semantics.

7. *Is your pluralistic stance related to the views of JC Beall and Greg Restall stated in Logical Pluralism (Beall and Restall, 2005)? According to these authors, the concept of logical consequence allows any specification that respect the so-called "Generalized Tarski Thesis", according to which an argument is valid if and only if, in every case in*

which the premises are true, so is the conclusion. The sort of pluralism they endorse thus admits as legitimate specifications of the relation of logical consequence such diverse conceptions as the modal conception based on possible worlds and the classical Tarskian conception based on models.

SS. What I find interesting about their approach is that it is a pluralism *within* the model-theoretic tradition. In my terms, I take them to be arguing that even the model-theoretic notion of logical consequence can be sharpened in various, mutually incompatible directions. Early in the book, Beall and Restall seem to make room for a different sort of pluralism, one that does not privilege the model-theoretic approach. I don't know if this is directed at a second sort of pluralism, one that privileges a proof-theoretic notion of consequence, or if the pluralism they suggest here accepts both a model-theoretic and an inferentialist approach, as good sharpenings of the intuitive notion of logical consequence. Frankly, the remark is too sketchy to be sure. It should be noted that Restall has recently articulated a proof-theoretic pluralism, in an article forthcoming in *The Monist*.

Later in the book, Beall and Restall insist that logical consequence is itself model-theoretic (as given by the generalized Tarski scheme). So, at that point, they seem to be rejecting a more far reaching pluralism. They strongly imply that proof-theoretic systems are out of bounds. This occurs when they turn their attention to non-transitive and non-reflexive logics. Those are rejected – as logics – since they cannot be fit into the generalized Tarski scheme. Presumably, the fact that at least some of these logics have proof-theoretic motivations is irrelevant.

8. *The abandonment of classical logic has been often invoked to make justice of a widespread phenomenon of natural languages: vagueness. Most of the words we use in natural languages give rise to borderline cases and their extensions seem to lack clear boundaries. In your book Vagueness in Context (Shapiro, 2006) you advance a novel theory of vagueness. The semantics you propose in the book is a novel application of some central ideas of supervaluationism merged in a contextualist framework (e.g. Fine 1975). Two central tenets of standard supervaluationism are that in borderline cases vague expressions can be precisified in more than one semantically legitimate way, and that borderline sentences are (super)true just in case they count as true in each of these legitimate precisifications: the mantra of standard supervaluationism is thus that truth is supertruth (truth in every precisification) and that borderline cases involve truth-value gaps. Your theory of vagueness drops this latter tenet of standard supervaluationism by distinguishing indeterminate truth from lack of truth-value. Could you say what are the main reasons that led you to think that this is the correct approach for vagueness?*

SS. One motivation for my view is the observation that vague predicates are used in a rather fluctuating way, especially in the borderline region. What counts as “red” or “bald” depends heavily on context. I take my thesis of “open-texture” to be empirically supported. A main impetus for my view was the pioneering work of my friend and then colleague, Diana Raffman (e.g. Raffman, 1996). My first goal was to provide a model-theoretic framework for her views on vagueness. But then my own ideas on open-texture took hold, and I developed them in their own right. In the end, I don’t think my own views are that much at odds with Raffman's. I find the source of variation (within the borderline region) to lie in the semantics and pragmatics of the use of vague terms. She looks to psychology. Formally, however, the systems are close.

I am also much taken by Friedrich Waismann's views on open-texture, and the dynamic view of language generally. Vagueness is but one instance of this, but it bears on most of the issues in the philosophy of language, the philosophy of logic, and, to some extent, the philosophy of mathematics.

9. *Philosophy of mathematics and logic have proved to be very lively areas in recent times. Are there any particular issues that you would suggest to students at their early stages as being those in which further research is currently needed, or those which are likely to lead to novel and promising debates in the near future?*

SS. That is hard to say. I very much admire the work you mentioned earlier, on the nature of actual mathematical practice, and the role of mathematics in the sciences. I am much interested in the nature of explanation within mathematics, and in the role of mathematics in ordinary, scientific explanation.

Another area of interest, for me at least, would be to tie the work in linguistics to the languages of mathematics. That is, I'd like to see the languages of mathematics as a focus of informed work in the philosophy of language, and in semantics generally. Do the languages of mathematics provide interesting case studies for the nature of language generally? Or are the languages of mathematics more an exception to the main features of natural languages.

10. *Let us close this interview with a more general look at the role of philosophy, and philosophy of mathematics in particular, in society. The philosophy of mathematics and the philosophy of logic are fairly specialized areas of inquiries. Some may perceive them -- especially the former -- as not so central to philosophy, and immaterial to the public profile that intellectuals are supposed to have, at least according to some traditions. Do you think that research in these fields can serve to illuminate other general questions in philosophy, or even play some role in spreading philosophy among the general public?*

SS. Yes, I do. To a large extent, mathematics provides a wealth of case studies for many of the issues in mainstream philosophy today. Issues of epistemology, normativity, modality, language. Of course, mathematics is special, and one should not focus on that exclusively, but it is a remarkably rich area for philosophical inquiry.
A related theme starts from the fact that mathematics seems to be a central core of just any attempt to rigorously understand just about any aspect of reality, from physics, to biology, to economics, to linguistics. What does this say about the nature of mathematics, and about the intellectual enterprise in general? Indeed, what does it say about us, the empirical inquirers?

11. *Italy, among many other countries, has been thoroughly affected by the recent economic crisis. Universities are being severely influenced by this situation, and Humanities, which are perceived as having little or no practical effects in the short run,*

have being suffering from cuts and shortage of fundings. Is this problem as pressing in the USA as it is in Italy and other European countries at this moment?

SS. It is a serious problem in the United States, but I do not know enough to compare our situation to yours. Given the diverse nature of American Universities, and the different ways they are funded, the impact is uneven. Some universities were devastated, and had to cut back on essential programs. Others were affected less, but none emerged unscathed.

12. *Do you have any suggestions on how Humanities could be helped out of this difficult situation?*

SS. Not really. When funding has to be cut for such basic needs as health care, early education, nutrition, social work, etc, it is hard to make a strong case for higher education, and for the Humanities in particular. This is not to say, of course, that higher education, and the Humanities, are unimportant. Only that in the present climate, it is hard to set priorities.

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